**Problem assignment 3**

*Due: Wednesday, September 25, 2019*

**Problem 1**

Variables inferred by forward checking

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| vars | x | y | z | w | u | t | v |
| domain | 0,1,2...9 | 0,1,2...9 | 0,1,2...9 | 0,1,2...9 | 0,1,2...9 | 0,1,2...9 | 0,1,2...9 |
| x = 2 | 2 | 0,1,2...9 | 2,5,8 | 0,1,2...9 | 0,1,2...9 | 0,1,2...9 | 0,1,2...9 |
| y = 0 | 2 | 0 | 5 | 0,1,2...9 | 0,1,2...9 | 0,1,2...9 | 0,1,2...9 |
| t = 0 | 2 | 0 | 5 | 0,1,2...9 | 0,1,2...9 | 0 | 0 |
| Infer w | 2 | 0 | 5 | 5,9 | 0,1,2...9 | 0 | 0 |
| Infer u | 2 | 0 | 5 | 9 | 0,3,6,9 | 0 | 0 |

For the infer of w, since u isn’t assigned, we only consider the arc between w and z.

Then, the constraints would be:

w mod 4 = z mod 4 = 1

so w = {5,9}

For the infer of u, the constraints would be:

u mod 3 = v mod 3 = 0

u mod 3 = w mod 3 = 5 mod 3 or 9 mod 3 = {0,2}

For the first equation, we get u = {0,3,6,9}, for the second equation, we get u = {0,2,3,5,6,8,9}, then u = {0,3,6,9} ∩ {0,2,3,5,6,8,9} = {0,3,6,9}

Then since u is inferred, we should infer w again. Now the constraint for w is:

W mod 4 = z mod 4 = 1 -> w = {5,9}

W mod 3 = u mod 3 = 0 -> w = {0,3,6,9}

So the w = {5,9} ∩ {0,3,6,9} = 9.

So, the reasonable variable groups inferred by forward checking is:

x = 2

y = 0

z = 5

w = 9

u = {0,3,6,9}

t = 0

v = 0

Variables inferred by arc consistency

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| vars | x | y | z | w | u | t | v |
| domain | 0,1,2...9 | 0,1,2...9 | 0,1,2...9 | 0,1,2...9 | 0,1,2...9 | 0,1,2...9 | 0,1,2...9 |
| x = 2 | 2 | 0,1,2...9 | 2,5,8 | 0,1,2...9 | 0,1,2...9 | 0,1,2...9 | 0,1,2...9 |
| y = 0  (check arc z->w) | 2 | 0 | 5 | 0,1,2,4,5,6,8,9 | 0,1,2...9 | 0,1,2...9 | 0,1,2...9 |
| t = 0  (check arc t->v, w->u, z->w) | 2 | 0 | 5 | 5,9 | 0,1,2...9 | 0 | 0 |
| (check arc v->u, w->u) | 2 | 0 | 5 | 5,9 | 0,3,6,9 | 0 | 0 |
| (check u->w) | 2 | 0 | 5 | 9 | 0,3,6,9 | 0 | 0 |

So, the reasonable variable groups inferred by forward checking is:

x = 2

y = 0

z = 5

w = 9

u = {0,3,6,9}

t = 0

v = 0

Problem 2

**Part a.**

Initial tour: [0, 58, 36, 53, 10, 18, 17, 2, 6, 42, 41, 19, 13, 11, 29, 4, 51, 44, 30, 15, 27, 40, 37, 22, 7, 31, 3, 54, 50, 48, 21, 23, 38, 9, 47, 33, 26, 16, 56, 43, 28, 59, 34, 52, 45, 24, 20, 8, 14, 35, 55, 46, 1, 32, 12, 5, 39, 49, 25, 57]

Initial tour distance: 306.43717

Initial temperature: 100

Number of tours tried: 100000

Number of tours accepted: 92435

The best tour found: [0, 5, 53, 32, 34, 15, 18, 33, 25, 36, 17, 3, 44, 40, 55, 20, 9, 14, 46, 2, 4, 59, 48, 10, 21, 38, 24, 26, 19, 35, 6, 52, 12, 37, 22, 8, 49, 41, 28, 13, 58, 31, 43, 57, 11, 51, 47, 23, 54, 56, 29, 7, 50, 1, 27, 45, 39, 30, 42, 16]

The best tour found distance: 103.65166

**Part b.**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Parameters** | | **Initial Tour Distance** | **Number of Tours tried** | **Number of Tours Accepted** | **Best tour Distance** |
| **Steps: 100000** | **Temperature: 10000** | ~~311.539122~~ | ~~100000~~ | ~~99838~~ | ~~242.586178~~ |
| **Temperature: 1000** | ~~300.557104~~ | ~~100000~~ | ~~98869~~ | ~~187.442838~~ |
| **Temperature: 100** | 281.221808 | 100000 | 92424 | 102.865312 |
| **Temperature: 1** | 304.905496 | 100000 | 8224 | **63.707482** |
| **Temperature: 0.1** | 293.5911 | 100000 | 3672 | **70.27154** |
| **Steps: 200000** | **Temperature: 10000** | ~~322.237354~~ | ~~200000~~ | ~~199742~~ | ~~214.168836~~ |
| **Temperature: 1000** | ~~315.761656~~ | ~~200000~~ | ~~197847~~ | ~~161.578628~~ |
| **Temperature: 100** | 325.083958 | 200000 | 184680 | 85.275408 |
| **Temperature: 1** | 334.881072 | 200000 | 15452 | **63.81284** |
| **Temperature: 0.1** | 278.214292 | 200000 | 7408 | **68.78746** |
| **Steps: 400000** | **Temperature: 10000** | ~~290.435064~~ | ~~400000~~ | ~~399416~~ | ~~207.303222~~ |
| **Temperature: 1000** | ~~301.722132~~ | ~~400000~~ | ~~395590~~ | ~~123.624858~~ |
| **Temperature: 100** | 290.907394 | 400000 | 369320 | 77.615786 |
| **Temperature: 1** | **327.964436** | **400000** | **31233** | **63.666698** |
| **Temperature: 0.1** | 289.172244 | 400000 | 14092 | **67.207378** |
| **Steps: 800000** | **Temperature: 10000** | ~~333.904144~~ | ~~800000~~ | ~~798804~~ | ~~165.260582~~ |
| **Temperature: 1000** | 307.826264 | 800000 | 791020 | 103.25354 |
| **Temperature: 100** | 307.21158 | 800000 | 738602 | 72.957344 |
| **Temperature: 1** | 299.86413 | 800000 | 62739 | **65.110992** |
| **Temperature: 0.1** | 312.247572 | 800000 | 28704 | **67.02021** |
| **Steps: 1600000** | **Temperature: 10000** | ~~283.527504~~ | ~~1600000~~ | ~~1597645~~ | ~~156.056872~~ |
| **Temperature: 1000** | 278.29668 | 1600000 | 1582088 | 85.168188 |
| **Temperature: 100** | 319.700394 | 1600000 | 1477070 | **71.719568** |
| **Temperature: 1** | 287.510608 | 1600000 | 124901 | **64.41229** |
| **Temperature: 0.1** | 305.709702 | 1600000 | 58653 | **63.677714** |

With the same steps, as initial temperature decreased, the best tour distance found would decrease in general.

With the same temperature, as the steps increased, the best tour distance found would decrease in general.

The best solution appears with the parameter: steps=400000, initial\_ temperature=1.

The best solution is 63.666698.

The red solutions are those beat the best solution in problem (72.892862).

**Part c.**

My cooling method for the temperature is:

Ti = Tinit - (Thalf / (0.3K)2) \* i2 0<i<0.3K

Ti = Thalf/(0.7K)2\*(i-K)2 0.3K≤i<K

Tinit is the initial temperature.

Thalf is half of the initial temperature.

K is the number of steps.

Generally, it’s two part of quadratic function.

10 tries with tours accepted over 20000:

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Initial distance | 322.813260 | 320.257628 | 309.813396 | 292.906218 | 281.33309 | 283.037410 | 314.996344 | 316.949372 | 288.46531 | 296.115862 |
| Best tour distance | 81.042122 | 82.351288 | 78.570942 | 79.272502 | 79.434684 | 79.663748 | 78.559722 | 83.073736 | 81.429768 | 82.892166 |
| Tours accepted | 20180 | 20245 | 20204 | 20169 | 20145 | 20131 | 20090 | 20115 | 20263 | 20329 |

Then, the average energy for the above 10 best distance are: 80.6290678

**Problem 3.**

number of generations range between 200-1000, population=500, mutation probability=0.05, culling=0.05, elite=0.05

|  |  |  |  |
| --- | --- | --- | --- |
|  | Start state distance | Best distance found | pics |
| N\_generations=200 | 248.48 | 105.17 | evolution_N_gen_200_N_pop_500_p_mut_05_p_cull_05_p_elite_05 |
| N\_generations=300 | 258.48 | 80.85 | evolution_N_gen_300_N_pop_500_p_mut_05_p_cull_05_p_elite_05 |
| N\_generations=400 | 253.62 | 78.43 | evolution_N_gen_400_N_pop_500_p_mut_05_p_cull_05_p_elite_05 |
| N\_generations=500 | 259.32 | 74.04 | evolution_N_gen_500_N_pop_500_p_mut_05_p_cull_05_p_elite_05 |
| N\_generations=600 | 258.34 | 75.14 | evolution_N_gen_600_N_pop_500_p_mut_05_p_cull_05_p_elite_05 |
| N\_generations=700 | 252.7 | 67.2 | evolution_N_gen_700_N_pop_500_p_mut_05_p_cull_05_p_elite_05 |
| N\_generations=800 | 248.47 | 64.99 | evolution_N_gen_800_N_pop_500_p_mut_05_p_cull_05_p_elite_05 |
| N\_generations=900 | 243.34 | 72.84 | evolution_N_gen_900_N_pop_500_p_mut_05_p_cull_05_p_elite_05 |
| N\_generations=1000 | 254.68 | 69.8 | evolution_N_gen_1000_N_pop_500_p_mut_05_p_cull_05_p_elite_05 |

number of generations=500, population range between 250-750, mutation probability=0.05, culling=0.05, elite=0.05

|  |  |  |  |
| --- | --- | --- | --- |
|  | Start state distance | Best distance found | pics |
| N\_pop=250 | 259.92 | 77.48 | evolution_N_gen_500_N_pop_250_p_mut_05_p_cull_05_p_elite_05 |
| N\_pop=300 | 260.95 | 72.07 | evolution_N_gen_500_N_pop_300_p_mut_05_p_cull_05_p_elite_05 |
| N\_pop=350 | 257.11 | 73.94 | evolution_N_gen_500_N_pop_350_p_mut_05_p_cull_05_p_elite_05 |
| N\_pop=400 | 250.02 | 77.72 | evolution_N_gen_500_N_pop_400_p_mut_05_p_cull_05_p_elite_05 |
| N\_pop=450 | 250.79 | 74.75 | evolution_N_gen_500_N_pop_450_p_mut_05_p_cull_05_p_elite_05 |
| N\_pop=500 | 255.39 | 70.79 | evolution_N_gen_500_N_pop_500_p_mut_05_p_cull_05_p_elite_05 |
| N\_pop=550 | 241.84 | 72.85 | evolution_N_gen_500_N_pop_550_p_mut_05_p_cull_05_p_elite_05 |
| N\_pop=600 | 251.63 | 70.52 | evolution_N_gen_500_N_pop_600_p_mut_05_p_cull_05_p_elite_05 |
| N\_pop=650 | 237.17 | 72.11 | evolution_N_gen_500_N_pop_650_p_mut_05_p_cull_05_p_elite_05 |
| N\_pop=700 | 249.25 | 70.76 | evolution_N_gen_500_N_pop_700_p_mut_05_p_cull_05_p_elite_05 |
| N\_pop=750 | 237.13 | 73.22 | evolution_N_gen_500_N_pop_750_p_mut_05_p_cull_05_p_elite_05 |

number of generations=500, population=500, mutation probability range between 0.00-0.25, culling=0.05, elite=0.05

|  |  |  |  |
| --- | --- | --- | --- |
|  | Start state distance | Best distance found | pics |
| mutation\_prob=0.00 | 250.23 | 79.17 | evolution_N_gen_500_N_pop_500_p_mut_00_p_cull_05_p_elite_05 |
| mutation\_prob=0.05 | 251.27 | 68.19 | evolution_N_gen_500_N_pop_500_p_mut_05_p_cull_05_p_elite_05 |
| mutation\_prob=0.10 | 240.41 | 74.95 | evolution_N_gen_500_N_pop_500_p_mut_10_p_cull_05_p_elite_05 |
| mutation\_prob=0.15 | 247.96 | 79.87 | evolution_N_gen_500_N_pop_500_p_mut_15_p_cull_05_p_elite_05 |
| mutation\_prob=0.20 | 254.75 | 97.73 | evolution_N_gen_500_N_pop_500_p_mut_20_p_cull_05_p_elite_05 |
| mutation\_prob=0.25 | 251.86 | 110.12 | evolution_N_gen_500_N_pop_500_p_mut_25_p_cull_05_p_elite_05 |

number of generations=500, population=500, mutation probability=0.05, culling range between 0.00-0.25, elite=0.05

|  |  |  |  |
| --- | --- | --- | --- |
|  | Start state distance | Best distance found | pics |
| culling\_perc=0.00 | 242.99 | 88.26 | evolution_N_gen_500_N_pop_500_p_mut_05_p_cull_00_p_elite_05 |
| culling\_perc=0.05 | 254.31 | 73.32 | evolution_N_gen_500_N_pop_500_p_mut_05_p_cull_05_p_elite_05 |
| culling\_perc=0.10 | 252.39 | 72.67 | evolution_N_gen_500_N_pop_500_p_mut_05_p_cull_10_p_elite_05 |
| culling\_perc=0.15 | 250.24 | 70.62 | evolution_N_gen_500_N_pop_500_p_mut_05_p_cull_15_p_elite_05 |
| culling\_perc=0.20 | 251.24 | 67.32 | evolution_N_gen_500_N_pop_500_p_mut_05_p_cull_20_p_elite_05 |
| culling\_perc=0.25 | 250.04 | 73.8 | evolution_N_gen_500_N_pop_500_p_mut_05_p_cull_25_p_elite_05 |

number of generations=500, population=500, mutation probability=0.05, culling=0.05, elite range between 0.00-0.25

|  |  |  |  |
| --- | --- | --- | --- |
|  | Start state distance | Best distance found | pics |
| elite\_perc =0.00 | 253.74 | 235.69 | evolution_N_gen_500_N_pop_500_p_mut_05_p_cull_05_p_elite_00 |
| elite\_perc =0.05 | 245.17 | 73.7 | evolution_N_gen_500_N_pop_500_p_mut_05_p_cull_05_p_elite_05 |
| elite\_perc =0.10 | 245.18 | 65.81 | evolution_N_gen_500_N_pop_500_p_mut_05_p_cull_05_p_elite_10 |
| elite\_perc =0.15 | 250.26 | 69.83 | evolution_N_gen_500_N_pop_500_p_mut_05_p_cull_05_p_elite_15 |
| elite\_perc =0.20 | 253.78 | 73.47 | evolution_N_gen_500_N_pop_500_p_mut_05_p_cull_05_p_elite_20 |
| elite\_perc =0.25 | 249.95 | 69.85 | evolution_N_gen_500_N_pop_500_p_mut_05_p_cull_05_p_elite_25 |

Analysis and conclusion:

As the number of generations increase, the best distance result is getting smaller till n\_generations=800, then the result would fluctuate around 70, which might mean the evolution of the population is meeting the bottle-neck. However, as the number of generations increase, the average distance tend to become more closer to the best distance, which means every single “person” in the whole population tend to become the best “person”.

About the population, it seems that there isn’t any specific relations between the population and the best distance found. That still make sense for GA. The population only presents a sample space for the whole process. Its size somehow doesn’t really matter.

The mutation probability is very interesting. Reasonable percentage of the mutation would increase the performance of the system. For out test, the mutation\_prob=0.05 seems to meet the best distance, then the results become worse. Also, as the mutation probability increases, the average distance would get far away from the best distance, since more mutations means instability of the “genotype” in the population.

About the culling percentage, the best distance result appears when culling\_perc=0.20. Thus, same to the mutation probability, the reasonable percentage of culling would make it closer to best distance.

And for elite percentage, something very funny happened when elite\_perc=0. The whole result would fluctuate severely, which means elite is necessary in genetic algorithm. Then, the best distance result is when elite\_perc=0.10. Also we need reasonable elite percentage.

Finally, for both culling and elite percentage, the higher the rate, the closer the average distances and best results are. That’s easy to understand. These two parameters both means to preserve the “good gene”, so as generation increases, the “genotype” would generally become as good as the best.